

# DECAY CONSTANTS OF THE PSEUDOSCALAR CHARMONIUM AND BOTTOMONIUM

Zhi-Gang Wang<sup>1</sup><sup>1</sup>, Wei-Min Yang<sup>2</sup> and Shao-Long Wan<sup>2</sup>

<sup>1</sup> Department of Physics, North China Electric Power University, Baoding 071003,  
P. R. China

<sup>2</sup> Department of Modern Physics, University of Science and Technology of China,  
Hefei 230026, P. R. China

## Abstract

In this article, we investigate the structures of the pseudoscalar charmonium and bottomonium in the framework of the coupled rainbow Schwinger-Dyson equation and ladder Bethe-Salpeter equation with the confining effective potential (infrared modified flat bottom potential). As the current masses are very large, the dressing or renormalization for the  $c$  and  $b$  quarks are tender, however, mass poles in the timelike region are absent. The Euclidean time fourier transformed quark propagator has no mass poles in the timelike region which naturally implements confinement. The Bethe-Salpeter wavefunctions for those mesons have the same type (Gaussian type) momentum dependence and center around zero momentum with spatial extension to about  $q^2 = 1\text{GeV}^2$  which happen to be the energy scale for Chiral symmetry breaking, the strong interactions in the infrared region result in bound states. The decay constants for those pseudoscalar heavy quarkonia are compatible with the values of experimental extractions and theoretical calculations.

PACS : 14.40.-n, 11.10.Gh, 11.10.St, 12.40.qq

**Key Words:** Schwinger-Dyson equation, Bethe-Salpeter equation, decay constant, confinement

## 1 Introduction

Heavy quarkonium, bound state of the heavy quark and antiquark, characterized by at least three widely separated energy scales: the hard scale (the mass  $m$  of the heavy quarks), the soft scale (the relative momentum of the heavy quark-antiquark  $|\mathbf{p}|$ ) and the ultrasoft scale (the typical kinetic energy of the heavy quark-antiquark  $E$ ), plays a special role in probing the strong interactions in both the perturbative and nonperturbative regions. By definition of the heavy quark,  $m$  is large in comparison with the typical hadronic scale  $\Lambda_{QCD}$ , the corresponding processes can be successfully described in perturbative quantum chromodynamics (QCD) due to the asymptotic freedom. However, the lower scales  $|\mathbf{p}|$  and  $E$ , which are responsible for the binding, can not be accessible by perturbation theory. The appearance of multiscales in the dynamics of the heavy quarkonium makes its quantitative study

---

<sup>1</sup>Corresponding author; E-mail,wangzgyiti@yahoo.com.cn.

extremely difficult, the properties of the bound states and their decays can provide powerful test for QCD in both the perturbative and nonperturbative regions.

The physicists propose many original approaches to deal with the long distance properties of QCD, such as Chiral perturbation theory [1], heavy quark effective theory [2], QCD sum rules [3], lattice QCD [4], perturbative QCD [5], coupled Schwinger-Dyson equation (SDE) and Bethe-Salpeter equation (BSE) method [6], nonrelativistic QCD [7], potential nonrelativistic QCD [8], etc. All of those approaches have both outstanding advantages and obvious shortcomings in one or other ways. The coupled rainbow SDE and ladder BSE have given a lot of successful descriptions of the long distance properties of the low energy QCD and the QCD vacuum (for example, Refs. [9, 10, 11, 12] , for recent reviews one can see Refs.[13, 14]). The SDE can naturally embody the dynamical symmetry breaking and confinement which are two crucial features of QCD, although they correspond to two very different energy scales [15, 16]. On the other hand, the BSE is a conventional approach in dealing with the two body relativistic bound state problems [17]. From the solutions of the BSE, we can obtain useful information about the under-structure of the mesons and obtain powerful tests for the quark theory. However, the obviously drawback may be the model dependent kernels for the gluon two-point Green's function and the truncations for the coupled divergent SDE and BSE series in one or the other ways[18]. Many analytical and numerical calculations indicate that the coupled rainbow SDE and ladder BSE with phenomenological potential models can give model independent results and satisfactory values [6, 9, 10, 11, 12, 13, 14]. The usually used effective potential models are confining Dirac  $\delta$  function potential, Gaussian distribution potential and flat bottom potential (FBP) [13, 14, 19, 20, 21]. The FBP is a sum of Yukawa potentials, which not only satisfies chiral invariance and fully relativistic covariance, but also suppresses the singular point that the Yukawa potential has. It works well in understanding the dynamical chiral symmetry breaking, confinement and the QCD vacuum as well as the meson structures, such as electromagnetic form factors, radius, decay constants [18, 22, 23].

During the past two years, the experiments have discovered a number of new states, for example, the  $\eta'_c$  in exclusive  $B \rightarrow KK_SK^-\pi^+$  decays by Belle [24] , the narrow  $D_{sJ}$  states by Babar, CLEO and Belle [25], evidence for the  $\Theta^+(1540)$  with quantum numbers of  $K^+n$  [26], and the  $X(3872)$  through decay to  $\pi^+\pi^-J/\psi$  by Belle [27]. New experimental results call for interpretations, offer opportunities to extend our knowledge about hadron spectrum and challenge our understanding of the strong interaction; furthermore, they revitalize the study of heavy quarkonia and stimulate a lot of theoretical analysis through the charmonia and bottomonia have been thoroughly investigated.

The decay constants of the pseudoscalar charmonium and bottomonium ( $\eta_c$  and  $\eta_b$ ) mesons play an important role in modern physics with the assumption of current-meson duality. The precise knowledge of the those values  $f_{\eta_c}$  and  $f_{\eta_b}$  will provide great improvements in our understanding of various processes convolving the  $\eta_c$

and  $\eta_b$  mesons, for example, the process  $B \rightarrow \eta_c K$ , where the mismatches between the theoretical and experimental values are large [28]. The  $\eta_c$  meson is already observed experimentally, the current experimental situation with the  $\eta_b$  meson is rather uncertain, yet the discovery of the  $\eta_b$  meson is one of the primary goals of the CLEO-c research program [29]; furthermore, the  $\eta_b$  meson may be observed in Run II at the Tevatron through the decay modes into charmed states  $D^* D^{(*)}$ [30]. It is interesting to combine those successful potential models within the framework of coupled SDE and BSE to calculate the decay constants of the pseudoscalar heavy quarkonia such as  $\eta_c$  and  $\eta_b$ . For previous studies about the electroweak decays of the pseudoscalar mesons with the SDE and BSE, one can consult Refs. [6, 9, 10, 11, 12, 13, 14]. In this article, we use an infrared modified flat-bottom potential (IMFBP) which takes the advantages of both the Gaussian distribution potential and the FBP to calculate the decay constants of those pseudoscalar heavy quarkonia.

The article is arranged as follows: we introduce the IMFBP in section II; in section III, IV and V, we solve the rainbow SDE and ladder BSE, explore the analyticity of the quark propagators, investigate the dynamical dressing and confinement, finally obtain the decay constants for those pseudoscalar heavy quarkonia; section VI is reserved for conclusion.

## 2 Infrared modified Flat Bottom Potential

The present techniques in QCD calculation can not give satisfactory large  $r$  behavior for the gluon two-point Green's function to implement the linear potential confinement mechanism , in practical calculation, the phenomenological effective potential models always do the work. As in our previous work [18], we use a gaussian distribution function to represent the infrared behavior of the gluon two-point Green's function,

$$4\pi G_1(k^2) = 3\pi^2 \frac{\varpi^2}{\Delta^2} e^{-\frac{k^2}{\Delta}}, \quad (1)$$

which determines the quark-antiquark interaction through a strength parameter  $\varpi$  and a range parameter  $\Delta$ . This form is inspired by the  $\delta$  function potential (in other words the infrared dominated potential) used in Refs.[19, 20], which it approaches in the limit  $\Delta \rightarrow 0$ . For the intermediate momentum, we take the FBP as the best approximation and neglect the large momentum contributions from the perturbative QCD calculations as the coupling constant at high energy is very small. The FBP is a sum of Yukawa potentials which is an analogy to the exchange of a series of particles and ghosts with different masses (Euclidean Form),

$$G_2(k^2) = \sum_{j=0}^n \frac{a_j}{k^2 + (N + j\rho)^2}, \quad (2)$$

where  $N$  stands for the minimum value of the masses,  $\rho$  is their mass difference, and  $a_j$  is their relative coupling constant. Due to the particular condition we take for

the FBP, there is no divergence in solving the SDE. In its three dimensional form, the FBP takes the following form:

$$V(r) = - \sum_{j=0}^n a_j \frac{e^{-(N+j\rho)r}}{r}. \quad (3)$$

In order to suppress the singular point at  $r = 0$ , we take the following conditions:

$$\begin{aligned} V(0) &= \text{constant}, \\ \frac{dV(0)}{dr} &= \frac{d^2V(0)}{dr^2} = \dots = \frac{d^nV(0)}{dr^n} = 0. \end{aligned} \quad (4)$$

The  $a_j$  can be determined by solve the equations inferred from the flat bottom condition Eq.(4). As in previous literature [18, 21, 22, 23],  $n$  is set to be 9. The phenomenological effective potential (IMFBP) can be taken as

$$G(k^2) = G_1(k^2) + G_2(k^2). \quad (5)$$

### 3 Schwinger-Dyson equation

The SDE can provide a natural framework for investigating the nonperturbative properties of the quark and gluon Green's functions. By studying the evolution behavior and analytic structure of the dressed quark propagators, we can obtain valuable information about the dynamical dressing phenomenon and confinement. In the following, we write down the rainbow SDE for the quark propagator,

$$S^{-1}(p) = i\gamma \cdot p + \hat{m}_{c,b} + 4\pi \int \frac{d^4k}{(2\pi)^4} \gamma_\mu \frac{\lambda^a}{2} S(k) \gamma_\nu \frac{\lambda^a}{2} G_{\mu\nu}(k-p), \quad (6)$$

where

$$S^{-1}(p) = iA(p^2)\gamma \cdot p + B(p^2) \equiv A(p^2)[i\gamma \cdot p + m(p^2)], \quad (7)$$

$$G_{\mu\nu}(k) = (\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})G(k^2), \quad (8)$$

and  $\hat{m}_{c,b}$  stands for the current quark mass that explicitly breaks chiral symmetry.

The full SDE for the quark propagator is a divergent series of coupled nonlinear integral equations for the propagators and vertexes, we have to make truncations in one or other ways. The rainbow SDE has given a lot of successful descriptions of the QCD vacuum and low energy hadron phenomena [6, 13, 14, 15, 16], in this article, we take the rainbow SDE. If we go beyond the rainbow approximation, the bare vertex  $\gamma_\mu \frac{\lambda^a}{2}$  has to be substituted by the full quark-gluon vertex  $\Gamma_\mu^a(qqg)$ , which satisfies the Slavnov-Taylor identity. In the weak coupling limit,  $g^2 \rightarrow 0$ , two Feynman diagrams contribute to the vertex  $\Gamma_\mu^a(qqg)$  at one-loop level due to the non-Abelian nature of QCD i.e. the self-interaction of gluons [31]. If we neglect the contributions from

the three-gluon vertex  $\Gamma_\mu^a(ggg)$  and retain an Abelian version, the vertex  $\Gamma_\mu^a(qqg)$  can be taken as  $\frac{\lambda^a}{2}\Gamma_\mu(qqp)$ , where the vertex  $\Gamma_\mu(qqp)$  is the quark-photon vertex which satisfies the Ward-Takahashi identity. In practical calculation, we can take the vertex  $\Gamma_\mu(qqp)$  to be the Ball-Chiu and Curtis-Pennington vertex [32, 33] so as to avoid solving the coupled SDE for the vertex  $\Gamma_\mu(qqp)$ . However, the nonperturbative properties of QCD at the low energy region suggest that the SDEs are strongly coupled nonlinear integral equations, no theoretical work has ever proven that the contributions from the vertex  $\Gamma_\mu^a(ggg)$  can be safely neglected due to the complex Dirac and tensor structures. The one Feynman diagram contributions version of the vertex  $\Gamma_\mu^a(qqg)$  i.e. neglecting the contributions from the vertex  $\Gamma_\mu^a(ggg)$  in dressing the vertex  $\Gamma_\mu^a(qqg)$  is inconsistent with the Slavnov-Taylor identity [31]. If we take the assumption that the contributions from the vertex  $\Gamma_\mu^a(ggg)$  are not different greatly from the vertex  $\Gamma_\mu^a(qqg)$ , we can multiply the contributions from the vertex  $\Gamma_\mu^a(qqg)$  by some parameters which effectively embody the contributions from the vertex  $\Gamma_\mu^a(ggg)$  [34].

In this article, we assume that a Wick rotation to Euclidean variables is allowed, and perform a rotation analytically continuing  $p$  and  $k$  into the Euclidean region. The Euclidean rainbow SDE can be projected into two coupled integral equations for  $A(p^2)$  and  $B(p^2)$ . Alternatively, one can derive the SDE from the Euclidean path-integral formulation of the theory, thus avoiding possible difficulties in performing the Wick rotation [35]. As far as only numerical results are concerned, the two procedures are equal. In fact, the analytical structures of quark propagators have interesting information about confinement, we will make detailed discussion about the  $c$  and  $b$  quarks propagators respectively in section V.

## 4 Bethe-Salpeter equation

The BSE is a conventional approach in dealing with the two body relativistic bound state problems [17]. The precise knowledge about the quark structures of the mesons will result in better understanding of their properties. In the following, we write down the ladder BSE for the pseudoscalar quarkonia,

$$S_+^{-1}(q + \frac{P}{2})\chi(q, P)S_-^{-1}(q - \frac{P}{2}) = \frac{16\pi}{3} \int \frac{d^4k}{(2\pi)^4} \gamma_\mu \chi(k, P) \gamma_\nu G_{\mu\nu}(q - k), \quad (9)$$

where  $S(q)$  is the quark propagator,  $G_{\mu\nu}(k)$  is the gluon propagator,  $P_\mu$  is the four-momentum of the center of mass of the pseudoscalar quarkonia,  $q_\mu$  is the relative four-momentum between the quark and antiquark,  $\gamma_\mu$  is the bare quark-gluon vertex, and  $\chi(q, P)$  is the Bethe-Salpeter wavefunction (BSW) of the bound state.

We can perform the Wick rotation analytically and continue  $q$  and  $k$  into the Euclidean region <sup>2</sup>. In the lowest order approximation, the BSW  $\chi(q, P)$  can be

---

<sup>2</sup>To avoid possible difficulties in performing the Wick rotation, one can derive the BSE from the Euclidean path-integral formulation of the theory.

written as

$$\chi(q, P) = \gamma_5 [iF_1^0(q, P) + \gamma \cdot P F_2^0(q, P) + \gamma \cdot q q \cdot P F_3^1(q, P) + i[\gamma \cdot q, \gamma \cdot P] F_4^0(q, P)]. \quad (10)$$

The ladder BSE can be projected into the following four coupled integral equations,

$$\sum_j H(i, j) F_j^{0,1}(q, P) = \sum_j \int_0^\infty k^3 dk \int_0^\pi \sin^2 \theta K(i, j), \quad (11)$$

the expressions of the  $H(i, j)$  and  $K(i, j)$  are cumbersome and neglected here.

Here we will give some explanations for the expressions of  $H(i, j)$ . The  $H(i, j)$ 's are functions of the quark's Schwinger-Dyson functions (SDF)  $A(q^2 + P^2/4 + q \cdot P)$ ,  $B(q^2 + P^2/4 + q \cdot P)$ ,  $A(q^2 + P^2/4 - q \cdot P)$  and  $B(q^2 + P^2/4 - q \cdot P)$ . The relative four-momentum  $q$  is a quantity in the Euclidean spacetime while the center of mass four-momentum  $P$  must be continued to the Minkowski spacetime i.e.  $P^2 = -m_{\eta_c, \eta_b}^2$ , this results in the  $q \cdot P$  varying throughout a complex domain. It is inconvenient to solve the SDE at the resulting complex values of the quark momentum, especially for the heavy quarks. As the dressing effect is minor, we can expand  $A$  and  $B$  in terms of Taylor series of  $q \cdot P$ , for example,

$$A(q^2 + P^2/4 + q \cdot P) = A(q^2 + P^2/4) + A(q^2 + P^2/4)'q \cdot P + \dots$$

The other problem is that we can not solve the SDE in the timelike region as the two point gluon Green's function can not be exactly inferred from the  $SU(3)$  color gauge theory even in the low energy spacelike region. In practical calculations, we can extrapolate the values of  $A$  and  $B$  from the spacelike region smoothly to the timelike region with suitable polynomial functions. To avoid possible violation with confinement in sense of the appearance of pole masses  $q^2 = -m^2(q^2)$  in the timelike region, we must be care in choosing the polynomial functions [20]. For the  $\eta_c$  meson, the mass is about  $3.0 GeV$ , the extrapolation to the timelike region with the quantity  $-m_{\eta_c}^2/4$  can be performed easily, however, the large mass of the  $\eta_b$  meson makes the extrapolation into the deep timelike region troublesome. Although the  $\eta_b$  meson has not been observed experimentally yet, the theoretical calculations indicate that its mass is about  $9.4 GeV$  [36]. As the dressed quark propagators comprise the notation of constituent quarks by providing a mass  $m(q^2) = B(q^2)/A(q^2)$ , which corresponding to the dynamical symmetry breaking phenomena for the light quarks. We can simplify the calculation greatly and avoid the problems concerning the extrapolations in solving the BSE by take the following propagator for the  $c$  and  $b$  quarks,

$$S^{-1}(q^2) = i\gamma \cdot q + M_{c,b}, \quad (12)$$

where the  $M_{c,b}$  is the Euclidean constituent quark mass with  $M_{c,b}^2 = m_{c,b}^2(q^2) = q^2$  obtained from the solution of the SDE Eq.(6).

Finally we write down the normalization condition for the BSW,

$$N_c \int \frac{d^4 q}{(2\pi)^4} Tr \left\{ \bar{\chi} \frac{\partial S_+^{-1}}{\partial P_\mu} \chi(q, P) S_-^{-1} + \bar{\chi} S_+^{-1} \chi(q, P) \frac{\partial S_-^{-1}}{\partial P_\mu} \right\} = 2P_\mu, \quad (13)$$

where  $\bar{\chi} = \gamma_4 \chi^+ \gamma_4$ .

## 5 Coupled rainbow SDE and ladder BSE and the decay constants

In this section, we explore the coupled equations of the rainbow SDE and ladder BSE for the pseudoscalar heavy quarkonia numerically, the final results for the SDFs and BSWs can be plotted as functions of the square momentum  $q^2$ .

In order to demonstrate the confinement of quarks, we have to study the analyticity of SDFs for the  $c$  and  $b$  quarks, and prove that there no mass poles on the real timelike  $q^2$  axial. In the following, we take the Fourier transform with respect to the Euclidean time  $T$  for the scalar part ( $S_s$ ) of the quark propagator [6, 13, 37],

$$S_s^*(T) = \int_{-\infty}^{+\infty} \frac{dq_4}{2\pi} e^{iq_4 T} \frac{B(q^2)}{q^2 A^2(q^2) + B^2(q^2)}|_{\vec{q}=0}, \quad (14)$$

where the 3-vector part of  $q$  is set to zero. If  $S(q)$  has a mass pole at  $q^2 = -m^2(q^2)$  in the real timelike region, the Fourier transformed  $S_s^*(T)$  would fall off as  $e^{-mT}$  for large  $T$  or  $\log S_s^* = -mT$ .

In our numerical calculations, for small  $T$ , the values of  $S_s^*$  are positive and decrease rapidly to zero and beyond with the increase of  $T$ , which are compatible with the result (curve tendency with respect to  $T$ ) from lattice simulations [38] ; for large  $T$ , the values of  $S_s^*$  are negative, except occasionally a very small fraction of positive values. The negative values for  $S_s^*$  indicate an explicit violation of the axiom of reflection positivity [39], in other words, the quarks are not physical observable i.e. confinement.

For the  $c$  and  $b$  quarks, the current masses are very large, the dressing or renormalization is tender and the curves are not steep which in contrast to the dynamical chiral symmetry breaking phenomenon for the light quarks,  $m_c(0)/\hat{m}_c \simeq 1.5$  and  $m_b(0)/\hat{m}_b \simeq 1.1$ , however, mass poles in the timelike region are absent. At zero momentum,  $m_c(0) = 1937\text{MeV}$  and  $m_b(0) = 5105\text{MeV}$ , while the Euclidean constituent quark masses  $M_c = 1908\text{MeV}$  and  $M_b = 5096\text{MeV}$ , which defined by  $M^2 = m^2(q^2) = q^2$ , are compatible with the constituent quark masses in the literature. From the plotted BSWs (see Fig.1 as an example), we can see that the BSWs for pseudoscalar mesons have the same type (Gaussian type) momentum dependence while the quantitative values are different from each other. Just like the lighter  $\bar{q}q$  and  $\bar{q}Q$  pseudoscalar mesons [18], the gaussian type BSWs center around zero momentum with spatial extension to about  $q^2 = 1\text{GeV}^2$  which happen to be the energy scale for Chiral symmetry breaking, the strong interactions in the infrared region result in bound states. Finally we obtain the values for the decay constants of those pseudoscalar mesons which are defined by

$$\begin{aligned} if_\pi P_\mu &= \langle 0 | \bar{q} \gamma_\mu \gamma_5 q | \pi(P) \rangle, \\ &= N_c \int Tr [\gamma_\mu \gamma_5 \chi(k, P)] \frac{d^4 k}{(2\pi)^4}, \end{aligned} \quad (15)$$

here we use  $\pi$  to represent the pseudoscalar mesons<sup>3</sup>,

$$f_{\eta_c} = 349 \text{ MeV}; \quad f_{\eta_b} = 287 \text{ MeV}, \quad (16)$$

which are compatible with the results from the experimental extractions and theoretical calculations,  $f_{\eta_c} = 335 \pm 75 \text{ MeV}(Exp)$  [40];  $f_{\eta_c} \approx 400 \text{ MeV}(Exp)$  [41];  $f_{\eta_c} = 420 \pm 52 \text{ MeV}$ ,  $f_{\eta_b} = 705 \pm 27 \text{ MeV}(Theor)$  [42];  $f_{\eta_c} = 292 \pm 25 \text{ MeV}(Theor)$  [43];  $f_{\eta_c} \approx 350 \text{ MeV}(Theor)$  [44];  $f_{\eta_c} = 300 \pm 50 \text{ MeV}(Theor)$  [45]. In calculation, the values of  $\hat{m}_c$  and  $\hat{m}_b$  are taken as the current quark masses,  $\hat{m}_c = 1250 \text{ MeV}$  and  $\hat{m}_b = 4700 \text{ MeV}$ ; the input parameters for the FBP are  $N = 1.0\Lambda$ ,  $V(0) = -11.0\Lambda$ ,  $\rho = 5.0\Lambda$  and  $\Lambda = 200 \text{ MeV}$ , which are determined in study of the  $\bar{q}q$  and  $\bar{q}Q$  pseudoscalar mesons [18]. In this article, the Euclidean constituent quark masses for the  $c$  and  $b$  quarks are taken in solving the BSE as the dressing is tender. We borrow some idea from the fact that the simple phenomenological model of Cornell potential (Coulomb potential plus linear potential) with constituent quark masses can give satisfactory mass spectrum for the heavy quarkonia<sup>4</sup> and take larger values for the strength parameter  $\varpi$  and range parameter  $\Delta$ , i.e.  $\varpi = 2.2 \text{ GeV}$  and  $\Delta = 2.9 \text{ GeV}^2$ , in the infrared region comparing with the corresponding ones used in Ref.[18]. Furthermore the masses of the pseudoscalar mesons are taken as input parameters. If we take the Euclidean constituent quark masses  $M_c = m_c(0)$  and  $M_b = m_b(0)$ , the decay constants for the  $\eta_c$  and  $\eta_b$  mesons change slightly,  $f_{\eta_c} = 357 \text{ MeV}$  and  $f_{\eta_b} = 289 \text{ MeV}$ .

## 6 Conclusion

In this article, we investigate the under-structures of the pseudoscalar heavy quarkonia  $\eta_c$  and  $\eta_b$  in the framework of the coupled rainbow SDE and ladder BSE with the confining effective potential (IMFBP). After we solve the coupled rainbow SDE and ladder BSE numerically, we obtain the SDFs and BSWs for the pseudoscalar heavy quarkonia  $\eta_c$  and  $\eta_b$ . As the current masses of the  $c$  and  $b$  quarks are very large, the dressing or renormalization for the SDFs is tender and the curves are not steep which in contrast to the explicitly dynamical chiral symmetry breaking phenomenon for the light quarks, however, mass poles in the timelike region are absent. We can simplify the calculation greatly and avoid the problems concerning the extrapolations in solving the BSE by making the substitution  $B(q^2) \rightarrow M$  and  $A(q^2) \rightarrow 1$ . The BSWs for the pseudoscalar heavy quarkonia have the same type (Gaussian type) momentum dependence while the quantitative values are different from each other. The gaussian type BSWs center around zero momentum with spatial extension to about  $q^2 = 1 \text{ GeV}^2$  which happen to be the energy scale for Chiral symmetry breaking, the strong interactions in the infrared region result in bound states. Our numerical results for the values of the decay constants of the

---

<sup>3</sup>Here we write down the  $N_c$  explicitly according to the normalization condition Eq.(13).

<sup>4</sup>For an excellent review of the potential models, one can consult Ref.[46].

pseudoscalar heavy quarkonia are compatible with the corresponding ones obtained from the experimental extractions and theoretical calculations. Once the satisfactory SDFs and BSFs for the pseudoscalar heavy quarkonia are known, we can use them to investigate a lot of important quantities involving the  $B$ ,  $\eta_c$  and  $\eta_b$  mesons.

## Acknowledgment

This work is supported by National Natural Science Foundation, Grant Number 10405009, and Key Program Foundation of NCEPU. The authors are indebted to Dr. J. He (IHEP) for numerous help, without him, the work would not be finished. One of the author (Z.G.Wang) would like to thank Dr. Gogohia for helpful discussion.

## References

- [1] J. Gasser and H. Leutwyler, Ann. Phys. **158** (1984) 142; Nucl. Phys. **B250** (1985) 465.
- [2] M. Neubert, Phys. Rep. **245** (1994) 259.
- [3] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. **B147** (1979) 385, 448.
- [4] R. Gupta, hep-lat/9807028, for an elementary introduction.
- [5] S. J. Brodsky and G. P. Lepage , Phys. Lett. **B87** (1979) 359 ; Phys. Rev. **D22** (1980) 2157 .
- [6] C. D. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. **33** (1994) 477.
- [7] G. T. Bodwin, E. Braaten and G. P. Lepage, Phys. Rev. **D51** (1995) 1125, Erratum-ibid. **D55** (1997) 5853.
- [8] A. Pineda and J. Soto, Nucl. Phys. Proc. Suppl. **64** (1998) 428; N. Brambilla, A. Pineda, J. Soto and A. Vairo, Nucl. Phys. **B566** (2000) 275; N. Brambilla, A. Pineda, J. Soto and A. Vairo, hep-ph/0410047.
- [9] Y. B. Dai, C. S. Huang and D. S. Liu, Phys. Rev. **D43** (1991) 1717; Y. B. Dai, Y. B. Ding, C. S. Huang and C. L. Wang, Commun. Theor. Phys. **18** (1992) 313.
- [10] P. Maris and C. D. Roberts, Phys. Rev. **C56** (1997) 3369; P. Maris, C. D. Roberts and P. C. Tandy, Phys. Lett. **B420** (1998) 267.
- [11] P. Maris and P. C. Tandy, Phys. Rev. **C60** (1999) 055214; P. Maris, Nucl. Phys. **A663** (2000) 621.

- [12] M. A. Ivanov, Yu. L. Kalinovsky and C. D. Roberts, Phys. Rev. **D60** (1999) 034018; M. A. Ivanov, Yu. L. Kalinovsky, P. Maris and C. D. Roberts, Phys. Lett. **B416** (1998) 29; M. A. Ivanov, Yu. L. Kalinovsky, P. Maris and C. D. Roberts, Phys. Rev. **C57** (1998) 1991.
- [13] C. D. Roberts and S. M. Schmidt, Prog. Part. Nucl. Phys. **45** (2000) S1.
- [14] C. D. Roberts, nucl-th/0304050; P. Maris and C. D. Roberts, Int. J. Mod. Phys. **E12** (2003) 297.
- [15] V. A. Miransky, Dynamical Symmetry Breaking in Quantum Field Theories, Word Scientific, 1993.
- [16] R. Alkofer and L. V. Smekal, hep-ph/0007355; C. S. Fischer and R. Alkofer, hep-ph/0301094.
- [17] E. E. Salpeter and H. A. Bethe, Phys. Rev. **84** (1951) 1232; N. Nakanishi, Suppl. Prog. Theor. Phys. **43** (1969) 1.
- [18] Z. G. Wang, W. M. Yang and S. L. Wan, Phys. Lett. **B584** (2004) 71; Z. G. Wang , W. M. Yang and S. L. Wan, Nucl. Phys. **A744** (2004) 156.
- [19] H. J. Munczek and A. M. Nemirovsky, Phys. Rev. **D28** (1983) 181.
- [20] H. J. Munczek and P. Jain, Phys. Rev. **D46** (1991) 438; P. Jain and H. J. Munczek, **D44** (1991) 1873; P. Jain and H. J. Munczek, **D48** (1993) 5403.
- [21] K. L. Wang and S. L. Wan, Phys. Rev. **D47** (1993) 2098.
- [22] Z. G. Wang, S. L. Wan and K. L. Wang, Phys. Lett. **B498** (2001) 195; Z. G. Wang and S. L. Wan, Phys. Lett. **B 536** (2001) 241; Z. G. Wang, J. Phys. **G28** (2002) 3007; Z. G. Wang, S. L. Wan and K. L. Wang, Commun. Theor. Phys. **35** (2001) 697; Chin. Phys. **10** (2001) 497; Chin. Phys. Lett. **18** (2001) 743.
- [23] S. L. Wan and K. L. Wang, J. Phys. **G22** (1996) 1287.
- [24] S.-K. Choi, et al (the Belle Collaboration), Phys. Rev. Lett. **89** (2002) 102001; Erratum-ibid. **89** (2002) 129901.
- [25] B. Aubert et al (BABAR Collaboration), Phys. Rev. Lett. **90** (2003) 242001; D. Besson et al (CLEO Collaboration), Phys. Rev. **D 68** (2003) 032002; P. Krokovny et al (BELLE Collaboration), Phys. Rev. Lett. **91** (2003) 262002.
- [26] LEPS Collaboration, Phys. Rev. Lett. **91** (2003) 012002 ; V. V. Barmin et al, Phys. Atom. Nucl. **66** (2003) 1715; CLAS collaboration, Phys. Rev. Lett. **91** (2003) 252001 ; Phys. Rev. Lett. **92** (2004) 032001 ; Erratum - Ibid. 92 (2004) 049902; SAPHIR Collaboration, Phys. Lett. **B572** (2003) 127 ; A. E. Asratyan et al, hep-ex/0309042. HERMES Collaboration, Phys. Lett. **B585** (2004) 213

; SVD Collaboration, hep-ex/0401024; M. Abdel-Bary et al, hep-ex/0403011; ZEUS Collaboration, hep-ex/0403051.

- [27] S.-K. Choi et al (Belle Collaboration), Phys. Rev. Lett. **91** (2003) 262001.
- [28] L. Li, Z. G. Wang and T. Huang, Phys. Rev. **D70** (2004) 074006.
- [29] H. Stock, hep-ex/0310021.
- [30] F. Maltoni and A. D. Polosa, hep-ph/0405082.
- [31] A. I. Davydychev, P. Osland and L. Saks, Phys. Rev. **D63** (2001) 014022; A. I. Davydychev , P. Osland and L. Saks, Nucl. Phys. Proc. Suppl. **89** (2000) 277; V. Gogohia, hep-th/0406064.
- [32] J. S. Ball and T. W. Chiu, Phys. Rev. **D22** (1980) 2542.
- [33] D. C. Curtis and M. R. Pennington, Phys. Rev. **D42** (1990) 4165; Phys. Rev. **D44** (1991) 536; Phys. Rev. **D46** (1992) 2663.
- [34] M. S. Bhagwat, A. Holl, A. Krassnigg, C. D. Roberts and P. C. Tandy, Phys. Rev. **C70** (2004) 035205; M. S. Bhagwat and P. C. Tandy, Phys. Rev. **D70** (2004) 094039 .
- [35] S. T. Stainsby and R. T. Cahill, Phys. Lett. **146A** (1990) 467.
- [36] B. A. Kniehl, A. A. Penin, A. Pineda, V. A. Smirnov and M. Steinhauser, Phys. Rev. Lett. **92** (2004) 242001; ALEPH Collaboration (A. Heister et al.), Phys. Lett. **B530** (2002) 56 (A review of the mass of the  $\eta_b$  meson from theoretical calculations).
- [37] P. Maris, Phys. Rev. **D52** (1995) 6087.
- [38] M. S. Bhagwat, M. A. Pichowsky, C. D. Roberts and P. C. Tandy, Phys. Rev. **C68** (2003) 015203.
- [39] J. Glimm and A. Jaffee, Quantum Physics. A Functional Point of View (Springer-Verlag, New York, 1981).
- [40] CLEO Collaboration (K.W. Edwards et al.), Phys. Rev. Lett. **86** (2001) 30.
- [41] Particle Data Group, Phys. Rev. **D54** (1996) 1.
- [42] D. S. Hwang and G. H. Kim, Z. Phys. **C76** (1997) 107.
- [43] G. Cvetic, C. S. Kim, G. L. Wang and W. Namgung, Phys. Lett. **B596** (2004) 84.

- [44] V. A. Novikov, L. B. Okun, M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Phys. Rept. **41** (1978) 1.
- [45] N. G. Deshpande and J. Trampetic, Phys. Lett. **B339** (1994) 270.
- [46] C. Quigg and J. L. Rosner, Phys. Rept. **56** (1979) 167.

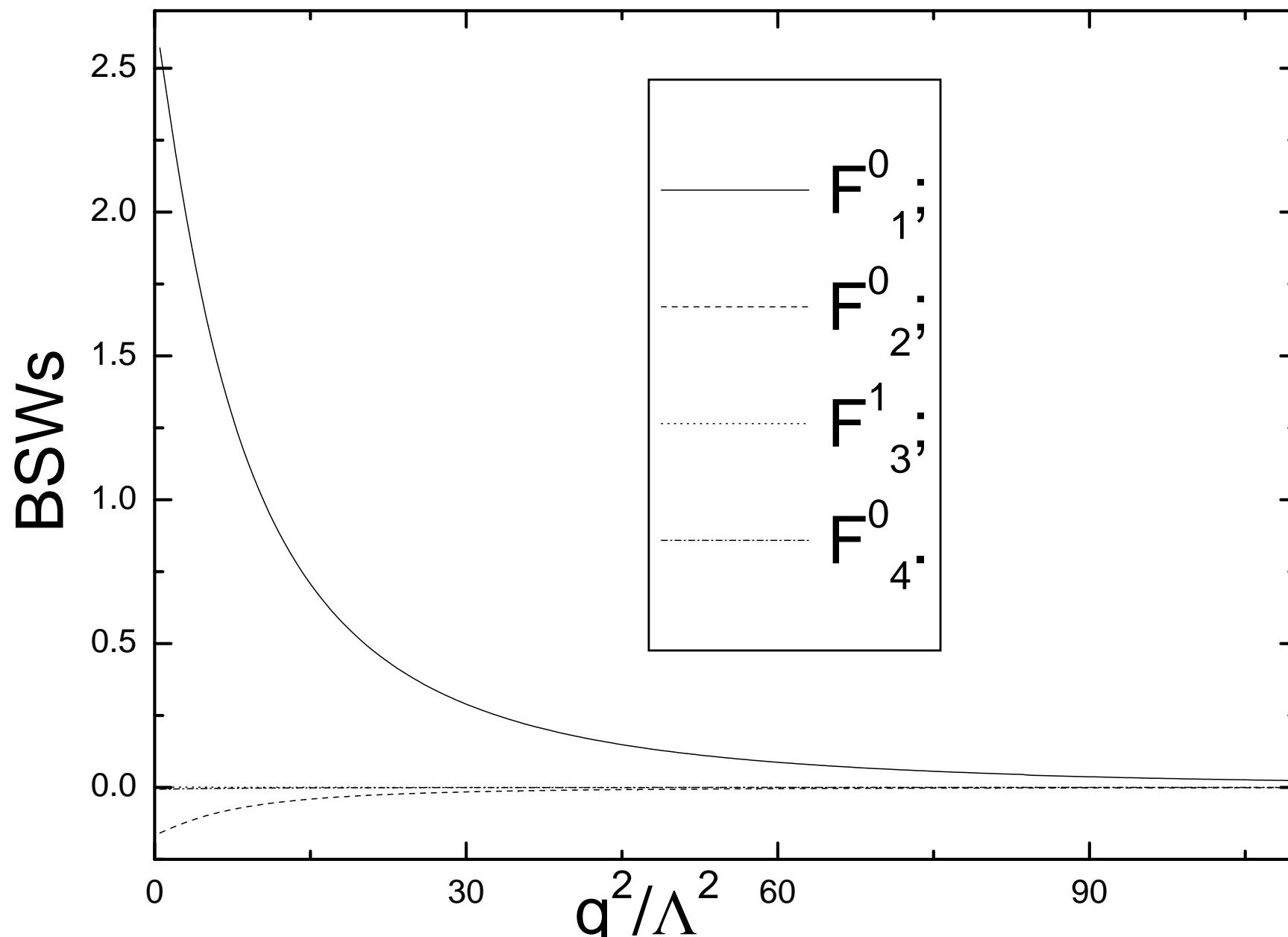


Fig.1 BSWs for charmonium